

PETAL GRAPHS

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Abstract: In this paper we introduce p -petal graphs. We prove that the necessary and sufficient condition for a planar p -petal graph G is that G has even number of petals each of size three. We also characterize the planar partial p -petal graphs.

AMS Subject Classification: 05C10

Key Words: petal graph, p -petal graph, partial p -petal graph, planarity

1. Introduction

A *petal graph* G is a simple connected (possibly infinite) graph with maximum degree three, minimum degree two, and such that the set of vertices of degree three induces a 2-regular graph G_{Δ} (possibly disconnected) and the set of vertices of degree two induces a totally disconnected graph G_{δ} . (see [1]). If G_{Δ} is disconnected, then each of its components is a cycle. In this paper, we consider petal graphs with a petals, P_0, P_1, \dots, P_{a-1} and r components, $G_{\Delta_0}, G_{\Delta_1}, \dots, G_{\Delta_{r-1}}$.

The vertex set of G is given by $V = V_1 \cup V_2$, where $V_1 = \{u_i\}$, $i = 0, 1, \dots, 2a - 1$ is the set of vertices of degree three, and $V_2 = \{v_j\}$, $j = 0, 1, \dots, a - 1$ is the set of vertices of degree two. The subgraph G_{δ} is the totally disconnected graph with vertex set V_2 . The subgraph G_{Δ} is the cycle $G_{\Delta} : u_0, u_1, \dots, u_{2a-1}$. The set $P(G) = P_0, P_1, \dots, P_{a-1}$ is the petal set

Received: July 7, 2011

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Abstract: In this paper we define (p_1, p_2, \dots, p_r) -petal graph and Petersen petal graph. We identify the petal graphs which are Petersen petal graphs. We also derive some results on the unit distance and domination number an of the Petersen petal graphs.

AMS Subject Classification: 05C10, 05C12, 05C69

Key Words: p -petal graph, (p_1, p_2, \dots, p_r) -petal graph, generalized Petersen graph, Petersen petal graph

1. Introduction

The concept of petal graphs was introduced in [1]. A petal graph is a simple connected (possibly infinite) graph G such that

1. $\Delta(G) = 3$;
2. $\delta(G) = 2$;
3. G_Δ is 2-regular (possibly disconnected);
4. Each edge of G is incident with at least one vertex in G_Δ .

Received: July 7, 2011

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Measuring the nonplanarity of p -petal graphs

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ABSTRACT

The necessary and sufficient condition for a p -petal graph to be planar is given in [1]. In this paper, we measure the nonplanarity of the p -petal graphs. Some results on crossing number, thickness, genus, and bisection width of p -petal graphs are proved.

Key words: p -petal graph, crossing number, thickness, genus, bisection width.

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INTRODUCTION

The planar graphs that can be drawn on a plane without edge crossings have a natural advantage for visualization. Many problems which cannot be solved for general graphs can be solved for planar graphs. Many results which are difficult to prove in case of general graphs can be proved not with much difficulty when restricted to planar graphs. The four color theorem could be one such problem.

The drawings will have the following properties:

- i. no edge passes through any other vertex than its end points;
- ii. no two edges touch each other, and no three edges cross at the same point.

In case of nonplanar graphs, our natural approach would be to draw them in a way as close as a planarity as possible. The question will be: how is a given graph far from being planar?

We can measure the nonplanarity of a graph on the whole, by using the following ideas:

- i. Crossing number;
- ii. Thickness;
- iii. Genus.

CROSSING NUMBER OF p -PETAL GRAPHS

The crossing number of G , denoted $CR(G)$, is the minimum number of edge crossings over all drawings of G . The study of crossing numbers began during the Second World War with Paul Turan's Brick Factory Problem, a problem of finding the crossing number of the complete bipartite graph. For details refer Turan[2]. Clearly, $CR(G) = 0$ if and only if G is planar.

Zarankiewicz[3] conjectured that the crossing number of the complete bipartite graph is $CR(K_{m,n}) = \lfloor \frac{m}{2} \rfloor \cdot \lfloor \frac{m-1}{2} \rfloor \cdot \lfloor \frac{n}{2} \rfloor \cdot \lfloor \frac{n-1}{2} \rfloor$. This result has been proved true for some special cases of the graph. Kleitman[4] proved the result when $\min\{m, n\} \leq 6$. Woodall [5] proved

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p

(p_1, p_2, \dots, p_r)

p

p

G

G_Δ

G_δ

G

G_Δ

$V = V_1 \cup V_2 \quad V_1 = \{u_i\}, i = 0, 1, \dots, 2a - 1$
 $V_2 = \{v_j\}, j = 0, 1, \dots, a - 1$

p

$G = P_{n,p}$

$P_{a+1} = P_0$

n

G

n

$\{P_j\}$

$l(P_i, P_{i+1}) = 2, i = 0, 1, 2, \dots, a - 1$

$G = P_{n,p}$

$p(G) = 1$

n

a

3

P^*

$P_{9,3}$

$p \geq 3$

P^*

$K_{3,3}$

a

3



